

# The spectrum and strong couplings of heavy-light hybrids

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## Abstract

The spectrum of the  $0^{++}$ ,  $0^{--}$ ,  $1^{-+}$  and  $1^{+-}$  heavy-light hybrids have been calculated in HQET. The interpolated current of the hybrid is chosen as  $g\bar{q}\gamma_\alpha G_{\alpha\mu}^a T^a h_v(x)$ ,  $g\bar{q}\gamma_\alpha\gamma_5 G_{\alpha\mu}^a T^a h_v(x)$  and  $g\bar{q}\sigma_{\mu\alpha} G_{\alpha\mu}^a T^a h_v(x)$ . Some strong couplings and decay widths of the heavy-light hybrids to  $B(D)\pi$  are calculated by using the QCD sum rules. The mass of  $0^{++}$  hybrid with gluon in TM( $1^{--}$ ) or TE( $1^{+-}$ ) mode is found similar, while their decay widths are different. A two-point correlation function between the pion and vacuum is employed and the leading order of  $1/M_Q$  expansion is kept in our calculation.

## 1 Introduction

It is almost twenty years to search the exotic hadrons such as the glueballs and hybrids. There are some special states which are regarded as the candidates of hybrids, especially the  $\hat{\rho}(1400)$  and  $\hat{\rho}(1600)$  have been studied widely, but no confirmation has been made so far. Recently, these two special states arouse great interest again. The E852 Collaboration at BNL[1] has reported a  $J^{pc} = 1^{-+}$  isovector resonance  $\hat{\rho}(1405)$  in the reaction  $\pi^- p \rightarrow \eta\pi^0 n$ . They also reported the mass  $1370 \pm 16^{+50}_{-30}$  MeV and width  $385 \pm 40^{+65}_{-105}$  MeV. The Crystal Barrel Collaboration has also claimed to find an evidence in  $p\bar{p}$  annihilation which may be resonance with a mass of  $1400 \pm 20 \pm 20$  MeV and a width of  $310 \pm 50^{+50}_{-30}$  MeV[1]. The confirmation of these states will provide some evidence for the existence of hybrids. At present, all the experiences specialize on the light quark hybrids for some reasons, but it's necessary to extend the energy region to hybrids including b or c quark, which is also possible in the B or  $\tau - C$  factory.

Theoretically, the spectrum and decay width of hybrids have been calculated widely with many methods such as bag model[2], flux-tube model[3], QCD sum rules[4], lattice [5] and other models[6]. However, there are few works about the spectrum and decay width of heavy-light hybrids[7]. HQET has led to much progress in the theoretical understanding of

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the properties of hadrons[8], one may ask a question whether it is suitable in the heavy-light hybrids. For the heavy-light hybrids, the sum rules' calculation in full QCD theory in Ref.[7] shows that the component of gluon gives a contribution more than  $1.0\text{GeV}$  to the mass, so the "light freedom" seems too heavy to keep the  $1/M_Q$  expansion available. The calculation of the spectrum of hybrids in HQET will give an answer to the question. Our results show that the spectrum of heavy-light hybrids including  $b$  or  $c$  quark are close to those in full QCD theory, it is suitable to deal with these hybrids within HQET. Besides, compared with  $\bar{b}bg$  and  $\bar{c}cg$  hybrid, the spectrum and decay width of the heavy-light hybrid is easier to be calculated in HQET.

It is interesting for the experimentalists to search the exotic  $1^{-+}$  heavy-light hybrids, so theoretical determination of the property of these states is necessary and urgent. In full QCD theory, the estimation in Ref.[7] showed that the sum rule for the mass of  $1^{-+}$  heavy-light hybrids had no platform at all. The masses of  $1^{-+}$  hybrids were given under the main assumption that the contribution of gluon condensate is less than 20% of the bare loop. In HQET, the ambiguous situation has been improved greatly. The surface of the  $\Lambda$  versus Borel variable  $\tau$  varies little in a large region, which gives a good platform and determines the masses of the hybrids.

According to the MIT bag model[9], the hybrids with the same  $J^{pc}$  have different internal interactions between the patrons which indicate that they are different states, that is to say, the gluon in hybrid can be in different TM( $1^{--}$ ) or TE( $1^{+-}$ ) mode. In order to predict the properties of them, we should choose suitable generating currents corresponding to these states for the calculation. In the case of light quark hybrids[4], these different  $0^{++}$  states were found to have different masses. In heavy-light hybrids case, the calculation shows that the mass split of the  $0^{++}$  heavy-light hybrids with gluon in different mode is not large in the  $1/M_Q \rightarrow \infty$  limit. The mass of  $0^{++}$  hybrids from two different currents,  $g\bar{q}\gamma_\alpha G_{\alpha\mu}^a T^a h_v(x)$  with gluon in TM( $1^{--}$ ) mode and  $g\bar{q}\sigma_{\mu\alpha} G_{\alpha\mu}^a T^a h_v(x)$  with gluon in TE( $1^{+-}$ ) mode, is found similar.

Though the mass of these two different  $0^{++}$  hybrids is similar, their decay widths to  $B(D)\pi$  final states are found different: the decay widths from current  $g\bar{q}\gamma_\alpha G_{\alpha\mu}^a T^a h_v(x)$  are about 86 MeV or 16 MeV corresponding to  $B\pi$  or  $D\pi$  final states, respectively, while the ones from current  $g\bar{q}\sigma_{\mu\alpha} G_{\alpha\mu}^a T^a h_v(x)$  are 11 MeV or 2.6 MeV.

The decay widths of the  $0^{++}$  and  $1^{-+}$  hybrids to  $B(D)\pi$  have been calculated in Ref.[10], where the decay constants of the hybrids were obtained from the formulae in full theory[7]. Both the strong couplings and the decay constants are calculated in HQET in this paper, and the strong coupling of the  $0^{++}$  hybrid obtained here is much larger than that in Ref.[10]. The large difference is from the values of the decay constant calculated in two ways.

In the calculation of the strong couplings, two-point function between the pion and vacuum is taken use of insteadly to avoid the ambiguity resulting from the double Borel transformation for the three-point correlation function and the infrared problem in soft pion limit. For convenience, the calculation is kept in the leading order of  $1/M_Q$  expansion.

The paper is organized as follows. The analytic formalism of HQET sum rules for the spectrum of hybrid is given in Sec. 2. In Sec. 3, We give the numerical results of the spectrum and decay constants of hybrid, the comparison of the spectrum with that in full QCD theory is given too. In Sec. 4, with the help of two-point correlation function between the pion and vacuum, the analytic formalism of HQET sum rules for the strong couplings

of hybrids is derived and the numerical results of some decay widths were obtained. In the last section, we give the conclusion and discussion.

## 2 HQET sum rules for the spectrum of the heavy-light hybrid mesons

As we know in Ref.[4], the gluon in hybrid can be in different mode, TM( $1^{--}$ ) or TE( $1^{+-}$ ) mode. To analyze these different hybrids, we must choose suitable generating current. For the spectrum of the  $0^{++}$  and  $1^{-+}$  heavy-light hybrids with the gluon in TM( $1^{--}$ ) and TE( $1^{+-}$ ) mode, respectively, the interpolated current in HQET is chosen as

$$j_\mu(x) = g\bar{q}\gamma_\alpha G_{\alpha\mu}^a T^a h_v(x). \quad (1)$$

where  $q(x)$  is the light quark field and  $h_v(x)$  is the heavy quark effective field,  $v$  is the velocity of the heavy quark.

Then, we construct the correlation function as

$$\begin{aligned} \Pi_{\mu\nu}(\omega) &= i \int d^4x e^{iqx} \langle 0 | T\{j_\mu(x), j_\nu^+(0)\} | 0 \rangle \\ &= (v_\mu v_\nu - g_{\mu\nu}) \Pi_v(\omega) + v_\mu v_\nu \Pi_s(\omega), \end{aligned} \quad (2)$$

where

$$\omega = 2q \cdot v. \quad (3)$$

Since the free heavy quark propagator in HQET is  $\int_0^\infty d\tau \delta(x - v\tau) \frac{1+\not{v}}{2}$  and the interaction of the heavy quark with the gluon field  $A_\mu$  in the leading order of  $1/M_Q$  expansion is  $g\bar{h}v \cdot Ah$ . Then under the fixed-point gauge  $x_\mu A_\mu = 0$  (which will be used throughout this paper), the full propagator of the heavy quark  $\langle 0 | T(h(x)\bar{h}(0)) | 0 \rangle$  in the leading order of  $1/M_Q$  expansion is the same as the free one. The freedom of heavy quark can be extracted out of the matrix element as a delta function, which facilitates the calculation.

In the operator product expansion, we keep the perturbative term, two gluon condensate, three gluon condensate and two quark condensate. The contribution of mixing condensate and higher dimension operators are negligible since their smallness. The Feynman diagrams are shown in Fig. 1, where the double line represents the propagator of the heavy quark. After twice suitable Borel transformation, we obtain the  $Im\Pi_s(\omega)$  and  $Im\Pi_v(\omega)$  corresponding to the scalar and vector contribution, respectively

$$\begin{aligned} Im\Pi_s(\omega) &= \frac{\alpha_s}{960\pi^2} \omega^6 + \frac{\alpha_s}{160\pi^2} m\omega^5 - \frac{1}{16} \langle \alpha_s G^2 \rangle \omega^2 - \frac{m}{8} \langle \alpha_s G^2 \rangle \omega \\ &\quad - \frac{\alpha_s}{6} \langle \bar{q}q \rangle \omega^3 + \frac{\alpha_s}{4} m \langle \bar{q}q \rangle \omega^2 - \frac{\alpha_s}{16} \langle gG^3 \rangle, \\ Im\Pi_v(\omega) &= \frac{\alpha_s}{960\pi^2} \omega^6 + \frac{\alpha_s}{480\pi^2} m\omega^5 + \frac{1}{48} \langle \alpha_s G^2 \rangle \omega^2 + \frac{m}{8} \langle \alpha_s G^2 \rangle \omega \\ &\quad - \frac{\alpha_s}{18} \langle \bar{q}q \rangle \omega^3 + \frac{\alpha_s}{4} m \langle \bar{q}q \rangle \omega^2 - \frac{\alpha_s}{48} \langle gG^3 \rangle, \end{aligned} \quad (4)$$

where the light quark mass corrections are considered also in these formulae.

As for the  $0^{--}$  and  $1^{+-}$  hybrids, the current was chosen as

$$j_{5\mu}(x) = g\bar{q}\gamma_\alpha\gamma_5 G_{\alpha\mu}^a T^a h_v(x). \quad (5)$$

The correlation function expanded is similar to the vector current case except for the opposite sign for the contribution of the quark condensates and their spectrum will be determined in a similar way.

For the  $0^{++}$  hybrid with the gluon in TE( $1^{+-}$ ) mode, the following current should be used to predict the mass

$$j(x) = g\bar{q}\sigma_{\mu\alpha}G_{\alpha\mu}^a T^a h_v(x). \quad (6)$$

The OPE of the  $\text{Im}\Pi(\omega)$  for this current has been carried out as

$$\text{Im}\Pi(\omega) = \frac{\alpha_s}{120\pi^2}\omega^6 - m\langle\alpha G^2\rangle\omega + 2\alpha_s m\langle\bar{q}q\rangle\omega^2 - \frac{\alpha}{16}\langle gG^3\rangle. \quad (7)$$

In the chiral limit, the two gluon and two quark condensates vanish.

On the phenomenal side, the decay constants of hybrids,  $F_{H^\pm}$ , are defined as

$$\langle 0|j_\mu|H(0^+)\rangle = F_{H^+}m_{H^+}^{1/2}v_\mu, \quad \langle 0|j_\mu|H(1^-)\rangle = F_{H^-}m_{H^-}^{1/2}\epsilon_\mu, \quad \langle 0|j|H'(0^+)\rangle = F'_{H^+}m_{H^+}'^{1/2}. \quad (8)$$

where  $m_H$  represent the masses of hybrids,  $\epsilon_\mu$  is the polarization vector and the two different  $0^{++}$  hybrids with gluon in TM( $1^{--}$ ) or TE( $1^{+-}$ ) mode is represented as  $H(0^{++})$  and  $H'(0^{++})$ , respectively. So the correlation function read

$$\begin{aligned} \Pi_s(\omega) &= -\frac{F_{H^+}^2}{(2\Lambda-\omega)} + \int_{\omega_c}^{\infty} d\omega' \frac{\text{Im}\Pi_s(\omega')}{\omega' - \omega}, \\ \Pi_v(\omega) &= -\frac{F_{H^+}^2}{(2\Lambda-\omega)} + \int_{\omega_c}^{\infty} d\omega' \frac{\text{Im}\Pi_v(\omega')}{\omega' - \omega}, \\ \Pi(\omega) &= -\frac{F_{H^+}'^2}{(2\Lambda-\omega)} + \int_{\omega_c}^{\infty} d\omega' \frac{\text{Im}\Pi(\omega')}{\omega' - \omega}. \end{aligned} \quad (9)$$

where the first term of the right side is the pole term resulting from lowest lying resonance contribution and the second term represents the contribution of the continuum state and higher resonances,  $\omega_c$  is the continuum threshold.

Taking use of the dispersion relations for the correlation function to equate the both sides, we obtain

$$\begin{aligned} \frac{F_{H^+}^2}{(2\Lambda-\omega)} &= -\frac{1}{\pi} \int_0^{\omega_c} d\omega' \frac{\text{Im}\Pi_s(\omega')}{\omega' - \omega}, \\ \frac{F_{H^-}^2}{(2\Lambda-\omega)} &= -\frac{1}{\pi} \int_0^{\omega_c} d\omega' \frac{\text{Im}\Pi_v(\omega')}{\omega' - \omega}, \\ \frac{F_{H^+}'^2}{(2\Lambda-\omega)} &= -\frac{1}{\pi} \int_0^{\omega_c} d\omega' \frac{\text{Im}\Pi(\omega')}{\omega' - \omega}. \end{aligned} \quad (10)$$

After the Borel transformation[11], they are turned into

$$F_{H^\pm}^2 e^{-2\Lambda/T} = -\frac{1}{\pi} \int_0^{\omega_c} d\omega' \text{Im}\Pi(\omega') e^{-\omega'/T}, \quad (11)$$

Table 1: *Masses of heavy-light hybrids with different  $J^{pc}$  (GeV).*

$J^{pc}$	$2\Lambda_c$	$2\Lambda_b$	$m_H(\bar{q}cg)$	$m_H(\bar{q}bg)$	$m_c(full)$	$m_b(full)$
$0^{++}$	4.4	4.4	3.5	6.9	4.0	6.8
$0^{--}$	6.8	6.8	4.7	8.1	4.5	7.7
$1^{-+}$	3.6	3.6	3.1	6.5	3.2	6.3
$1^{+-}$	3.8	3.8	3.2	6.6	3.4	6.5
$0^{++}$	4.2	4.2	3.4	6.8	none	none

where  $T$  is the Borel transformation variable. So the  $\Lambda$  can be determined as

$$2\Lambda = \frac{\int_0^{\omega_c} d\omega' \omega' Im\Pi(\omega') e^{-\omega'/T}}{\int_0^{\omega_c} d\omega' Im\Pi(\omega') e^{-\omega'/T}} \quad (12)$$

After the  $\Lambda$  has been calculated, the decay constant can be carried out according to (10).

### 3 Numerical results of the spectrum and decay constants of the hybrids

In this content, we will give the numerical results of the spectrum and decay constants of the hybrids. To proceed the process, the mass of the b and c quark are chosen as 4.7 GeV and 1.3 GeV, respectively, the condensates are chosen as

$$\begin{aligned} \langle 0 | m\bar{q}q | 0 \rangle &= -(0.1 GeV)^4, & \langle 0 | \bar{q}q | 0 \rangle &= -(0.24 GeV)^3, \\ \langle 0 | \alpha_s G^2 | 0 \rangle &= 0.06 GeV^4, & \langle 0 | gG^3 | 0 \rangle &= (0.27 GeV^2) \langle \alpha_s G^2 \rangle. \end{aligned} \quad (13)$$

and the scale of running coupling is set at the Borel parameter  $T$ .

The continuum threshold is chosen as below in the calculation:  $\omega_c = 5.0$  GeV for the  $0^{--}$  and two  $0^{++}$  hybrids from current  $j_\mu(x)$  and  $j(x)$ , and  $\omega_c = 4.5$  GeV for the  $1^{+-}$  and  $1^{-+}$  hybrids.

We display our results in HQET and those in full QCD theory in table. 1. In this table, the right two columns represent the mass of heavy-light hybrids calculated in full theory, the left represent the estimation in HQET and the bottom of this table represents the  $0^{++}$  hybrids with the gluon in TE( $1^{+-}$ ) mode.

From the table, the mass of hybrids including b or c quark in HQET is found similar to that in full theory, the light freedom in hybrids is not large enough to break down the  $1/M_Q$

Table 2: *Decay constants of heavy-light hybrids (  $GeV^{7/2}$  ).*

hybrid	$F_{H^+}(TM)$	$F_{H^-}$	$F'_{H^+}(TE)$
<i>c</i> quark	0.31	0.28	0.97
<i>b</i> quark	0.33	0.29	1.01

expansion. So the calculation in HQET is suitable, which implies that the  $1/M_Q$  correction to the sum rules is not large.

In  $1^{-+}$  hybrid case, the sum rule in full theory does not stabilize[7], which may indicate that no resonance exists in the channel. The masses of these states were given under the main assumption that the gluon condensate contribution is less than 20% of the bare loop. In HQET, the  $2\Lambda$  of  $1^{-+}$  hybrids versus Borel variable  $\tau$  vary little in a large region, which is shown in Fig. 2. The doted line represents that of *b* quark hybrid and the real line represents that of *c* quark, the little difference between them comes from the running coupling. The improvement of the ambiguity may come from the reason[11] that the  $T$ ,  $2\Lambda$  and  $\omega_c$  become constant low-energy parameters in the  $M_Q \rightarrow \infty$  limit in HQET, while the dependence of the parameters  $M^2$  and  $s_c$  on the heavy quark mass is *priori* not determined in full theory. Besides, the assumption in Ref.[7] is proved reasonable here.

In the case of light quark hybrids case, the mass of  $0^{++}$  hybrids with the gluon in different mode was found to have a large difference[4]. However, the  $\Lambda$  for the  $0^{++}$  heavy-light hybrids with the gluon in different mode is found similar in the  $M_Q \rightarrow \infty$  limit. The mass of the heavy-light hybrids in HQET is represented approximately

$$m \approx M_Q + \Lambda + O(1/M_Q), \quad (14)$$

so the mass split of the  $0^{++}$  heavy-light hybrids is not large in HQET. The mass of  $0^{++}$  hybrid with gluon in TM( $1^{--}$ ) mode is about 6.9 GeV and 3.5 GeV corresponding to *b* or *c* quark hybrid, respectively, and the mass of  $0^{++}$  hybrid with gluon in TE( $1^{+-}$ ) mode is about 6.8 GeV and 3.4 GeV, respectively.

When the radiative effects are taken account of, the effective current would receive renormalization improvement and the heavy quark expansion of the full current is necessary. However, in our derivation, neither the radiative effects nor the  $1/M_Q$  correction is taken account into.

The decay constants of the hybrids defined above can be obtained through formula.(10), they are all collected in table. 2. The table shows that the decay constants of the  $0^{++}$  hybrid with gluon in TE( $1^{+-}$ ) mode are larger than those with gluon in TM( $1^{--}$ ) mode.

## 4 Strong couplings and decay widths of heavy-light hybrids

In Ref.[10], we have calculated the decay width of

$$H_b(0^{++})(k) \longrightarrow B(0^{-+})(k - q) + \pi^\pm(q), \quad (15)$$

$$H_b(1^{-+})(k) \longrightarrow B(0^{-+})(k - q) + \pi^\pm(q), \quad (16)$$

where the  $0^{++}$  and  $1^{-+}$  hybrids with gluon in the TM( $1^{--}$ ) mode and TE( $1^{+-}$ ) mode, respectively, and the two-point correlation function results from current  $g\bar{q}\gamma_\alpha G_{\alpha\mu}^a T^a h_v(x)$ . The electric charges of the mesons except for pion have not been written out explicitly. The cases of  $H_c(0^{++}) \rightarrow D\pi^\pm$  and  $H_c(1^{-+}) \rightarrow D\pi^\pm$  have also been calculated there.

In this section, we will re-consider the same process in HQET firstly. For the decay widths of these processes, it is usually calculated through the three-point vertex function or QCD light-cone sum rules. However, in order to avoid the ambiguity of the three-point function resulted from the double Borel transformation and the infrared divergence in the soft pion approximation, we use the following two-point correlator between pion and vacuum

$$A_\nu(\omega', \omega, v) = i \int dx e^{ikx} \langle \pi^\pm(q) | T\{j_{1\nu}(x), j_2(0)\} | 0 \rangle = A(\omega', \omega)v_\nu + B(\omega', \omega)(-q_\nu + q \cdot v v_\nu) \quad (17)$$

where  $j_{1\nu}(x) = g\bar{q}\gamma_\mu G_{\mu\nu}^a T^a h_v(x)$ ,  $j_2(x) = \bar{h}_v \gamma_5 q(x)$ .  $A(\omega', \omega)$  and  $B(\omega', \omega)$  are scalar functions of  $\omega = 2k \cdot v$  and  $\omega' = 2(k - q) \cdot v$ , which are determined through the spectral density saturated by the mesons corresponding to the interpolated currents, respectively. The detailed OPE expansion of the  $A(\omega', \omega)$  and  $B(\omega', \omega)$  has been carried out in Ref.[10].

For the the  $0^{++}$  hybrid with gluon in TE( $1^{+-}$ ) mode, the current  $j_{1\nu}(x)$  in the correlation function above should be replaced by  $j'_1(x) = g\bar{q}\sigma_{\mu\alpha} G_{\alpha\mu}^a T^a h_v(x)$ . Then for the processes (15), we have another correlation function

$$C(\omega', \omega) = i \int d^4x e^{ikx} \langle \pi^\pm(q) | T\{j'_1(x), j_2(0)\} | 0 \rangle. \quad (18)$$

In the infinite heavy quark mass limit, the following approximate relation

$$2\Lambda - 2\Lambda' \approx \omega - \omega' = 2q \cdot v, \quad (19)$$

will be used in this paper. where  $\Lambda \sim m_H - M_Q$  and  $\Lambda' \sim m_{meson} - M_Q$ . Taking into account both the single pole terms and the double pole term in the physical side, we can express  $A(\omega', \omega)$ ,  $B(\omega', \omega)$  and  $C(\omega', \omega)$  functions of the single variable  $\omega'$ , respectively

$$A(\omega') = \frac{F_{H^+} f_m g_{H^+ m\pi} m_{H^+}^{1/2} m_m^2}{(2\Lambda' - \omega')^2 M_Q^3} + \frac{c_0}{2\Lambda' - \omega'}, \quad (20)$$

$$B(\omega') = \frac{F_{H^-} f_m g_{H^- m\pi} m_{H^-}^{1/2} m_m^2}{(2\Lambda' - \omega')^2 M_Q^3} + \frac{c_1}{2\Lambda' - \omega'}, \quad (21)$$

$$C(\omega') = \frac{F'_{H^+} f_m g'_{H^+ m\pi} m_{H^+}^{1/2} m_m^2}{(2\Lambda' - \omega')^2 M_Q^3} + \frac{c_2}{2\Lambda' - \omega'}, \quad (22)$$

where  $c_0$ ,  $c_1$  and  $c_2$  are constants.  $F_i$  are decay constants of hybrids defined above.  $f_m$  are decay constants of the B or D meson and  $g_{H^\pm m\pi}$  refers to strong couplings, they are defined as below

$$\begin{aligned}\langle 0|j_D|D\rangle &= -if_D m_D^2/M_c, \quad \langle 0|j_B|B\rangle = -if_B m_B^2/M_b, \\ \langle \pi^\pm(q)D|H'(0^{++})\rangle &= g'_{H^+ D\pi}, \quad \langle \pi^\pm(q)B|H'(0^{++})\rangle = g'_{H^+ B\pi}, \\ \langle \pi^\pm(q)D|H(0^{++})\rangle &= g_{H^+ D\pi}, \quad \langle \pi^\pm(q)D|H(1^{-+})\rangle = g_{H^- D\pi} \epsilon \cdot q, \\ \langle \pi^\pm(q)B|H(0^{++})\rangle &= g_{H^+ B\pi}, \quad \langle \pi^\pm(q)B|H(1^{-+})\rangle = g_{H^- B\pi} \epsilon \cdot q.\end{aligned}\quad (23)$$

The formulae (20) and (22) is a little different from those in Ref.[10] because of the different definition of the decay constants of the hybrids.

Taking use of the dispersion relation and making Borel transformation on them, we will get some equation about the strong couplings. After eliminating the  $c_i$  terms with appropriate differentiation, these strong couplings are obtained

$$\begin{aligned}g_{H^+ m\pi} &= \frac{M_Q^3}{F_{H^+} f_m m_{H^+}^{1/2} m_m^2} [2\Lambda' A'(\tau) + A_0] e^{2\Lambda'/\tau}, \\ g_{H^- m\pi} &= \frac{M_Q^3}{F_{H^-} f_m m_{H^-}^{1/2} m_m^2} [2\Lambda' B'(\tau) + B_0] e^{2\Lambda'/\tau}, \\ g'_{H^+ m\pi} &= -\frac{M_Q^3}{F'_{H^+} f_m m_{H^+}^{1/2} m_m^2} e^{2\Lambda'/\tau} [2\Lambda' C(\tau) + C_0],\end{aligned}\quad (24)$$

where  $A_0$  and  $B_0$  have been given in Ref.[10], while  $C(\tau)$  and  $C_0$  have the form as

$$C(\tau) = 8\sqrt{2}\{3[(m_H - m_m)b_2 - 2b_1] - (m_H - m_m)F_1/\tau\}, \quad (25)$$

$$C_0 = -8\sqrt{2}(m_H - m_m)F_1, \quad (26)$$

where the parameters  $b_i$ ,  $F_i$  have been calculated in Ref.[10] too.

Before going on the numerical calculation of the strong couplings and decay widths, it is necessary to fixing the parameters firstly. The masses of the heavy quarks and heavy mesons have been given in Ref.[12], the decay constants of B and D mesons are chosen as Ref.[13]. The masses and decay constants of the heavy-light hybrids have been computed above. The numerical results of the strong couplings are shown as *Fig.3 → Fig.5*, where the value of them is determined around  $\tau \sim 3.0 GeV$ . They are all displayed in table. 3, where the  $g_{H^- m\pi}$  is dimensionless.

To the processes (15) and (16), the decay widths are given by the following formulae

$$\begin{aligned}\Gamma(H(0^{++}) \rightarrow m(0^{-+}) + \pi) &= \frac{g_{H m\pi}^2 |q|}{8\pi m_H^2} = \frac{g_{H m\pi}^2 m_H^2 - m_m^2}{16\pi m_H^3}, \\ \Gamma(H(1^{-+}) \rightarrow m(0^{-+}) + \pi) &= \frac{g_{H m\pi}^2 |q|^3}{24\pi m_H^2} = \frac{(m_H^2 - m_m^2)^3 g_{H m\pi}^2}{192\pi m_H^5}.\end{aligned}\quad (27)$$

Then in the  $M_Q \rightarrow \infty$ , the numerical results of the decay widths read

$$\Gamma(H(0^{++}) \rightarrow B(0^{-+}) + \pi) = 86 MeV, \quad \Gamma(H(0^{++}) \rightarrow D(0^{-+}) + \pi) = 16 MeV, \quad (28)$$

$$\Gamma(H(1^{-+}) \rightarrow B(0^{-+}) + \pi) = 2.2 MeV, \quad \Gamma(H(1^{-+}) \rightarrow D(0^{-+}) + \pi) = 1.0 MeV. \quad (29)$$

$$\Gamma(H'(0^{++}) \rightarrow B(0^{-+}) + \pi) = 11 MeV, \quad \Gamma(H'(0^{++}) \rightarrow D(0^{-+}) + \pi) = 2.6 MeV. \quad (30)$$

Table 3: *Some parameters input and strong couplings of hybrids. ( GeV ).*

hybrid	$M_Q$	$m_m$	$f_m$	$m_H(0^{++})$	$m_H(1^{-+})$	$m'_H(0^{++})$	$g_{H+m\pi}$	$g'_{H+m\pi}$	$g_{H-m\pi}$
b	4.7	5.28	0.18	6.9	6.5	6.6	8.5	3.2	2.8
c	1.3	1.87	0.19	3.5	3.1	3.2	2.0	0.8	0.8

Though the decay of hybrids appears to follow the  $S + P$  selection rule, which means that the decay of hybrids to two S-wave mesons are suppressed [14], the selection rule is not absolute. In the flux tube and constituent glue models, it can be broken by wave function and relativistic effects, and the bag model predict that it is also possible that the excited quark loses its angular momentum to orbital angular momentum [15], the results obtained here support this idea.

The decay widths of the processes to  $B(D)\pi$  final states for  $H(0^{++})$  with gluon in TM( $1^{--}$ ) mode is much larger than those for  $H(1^{-+})$  with gluon in TE( $1^{+-}$ ) mode, the reason is that the final states in the later channels are in the P wave. Besides, the decay width of the  $H(0^{++}) \rightarrow B\pi$  obtained here is much larger than that we got in Ref.[10], the difference is from the decay constant. The decay constant of the  $0^{++}$  hybrid we got there in full theory is much smaller than the one calculated above in HQET.

Though the mass of these two  $0^{++}$  hybrids is almost the same, the strong couplings to pion of them are different, so the decay widths of these two different  $0^{++}$  hybrids are different. The decay width of the  $H'(0^{++}) \rightarrow B(D)\pi$  are smaller than those corresponding to  $H(0^{++}) \rightarrow B(D)\pi$ . The physical reason about the difference of decay width of these two different  $0^{++}$  hybrids is unknown to us yet, but the difference between these states provides a nice evidence that the decay property of these two  $0^{++}$  hybrids with the gluon in different mode is different.

## 5 Conclusion and Discussion

We calculate the spectrum of the  $0^{++}$ ,  $0^{--}$ ,  $1^{-+}$  and  $1^{+-}$  heavy-light hybrids with different currents in HQET, the results from current  $g\bar{q}\gamma_\alpha G_{\alpha\mu}^a T^a h_v(x)$  and  $g\bar{q}\gamma_\alpha\gamma_5 G_{\alpha\mu}^a T^a h_v(x)$  are compatible to those in full QCD theory. The calculation shows that the light freedom in heavy-light hybrids is not heavy enough to break down the  $1/M_Q$  expansion and it is suitable to apply HQET to heavy-light hybrid systems.

The sum rules for the masses of  $1^{-+}$  heavy-light hybrids have no platform at all in full theory, so the masses of them were given under some assumptions. The ambiguity of these sum rules have been improved in HQET, which suggests the reasonableness of the assumptions in Ref.[7] in another way. In the calculation, the leading order  $1/M_Q$  expansion approximation is used and only the first two terms in OPE are kept in our estimate, which will bring some errors. Besides, the decay constants will bring in large deviation too, it is

necessary to determine them more precisely.

Since the gluon in hybrids can be in different mode, the hybrids with the same  $J^{pc}$  but different gluon mode are in fact different states. In the case of light quark hybrids, the two different  $0^{++}$  states have different masses definitely. In the heavy-light hybrids case, the masses of these two different states are found similar in the  $M_Q \rightarrow \infty$  limit, however, the calculation shows that the decay widths in the processes of these two hybrids to  $B(D)\pi$  final states are different. The decay width of  $H(0^{++}) \rightarrow B(D)\pi$  is found about 86(16) MeV, while the decay width of  $H'(0^{++}) \rightarrow B(D)\pi$  is only 11(2.6) MeV.

The strong couplings and decay widths of the heavy-light hybrids in the processes of  $H(1^{-+}) \rightarrow B(D)\pi$  are calculated too. The large difference of the decay widths calculated here from those calculated in Ref.[10] lies on the large difference between the decay constants calculated in two different ways.

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# Figure caption

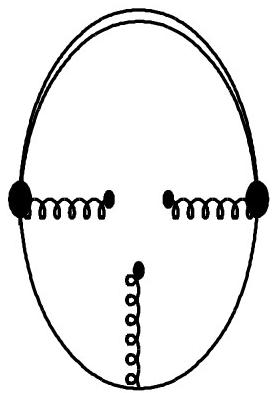
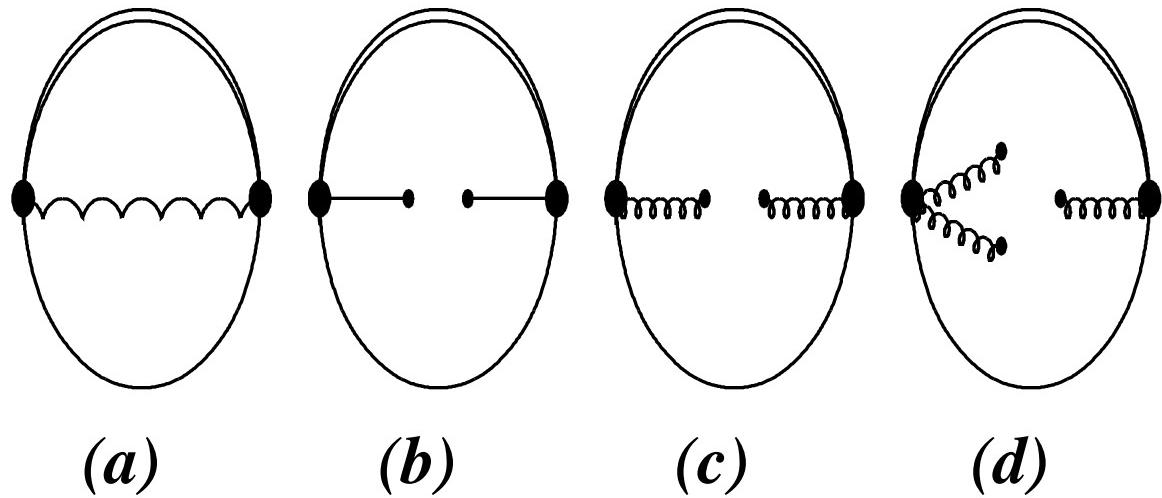
Fig. 1: Feynman diagrams contributing to the correlation function in HQET.

Fig. 2:  $2\Lambda$  of the  $1^{-+}$  heavy-light hybrids versus Borel variable  $T$ .

Fig. 3: Strong coupling of  $H(0^{++})$  heavy-light hybrids versus Borel variable  $\tau$ .

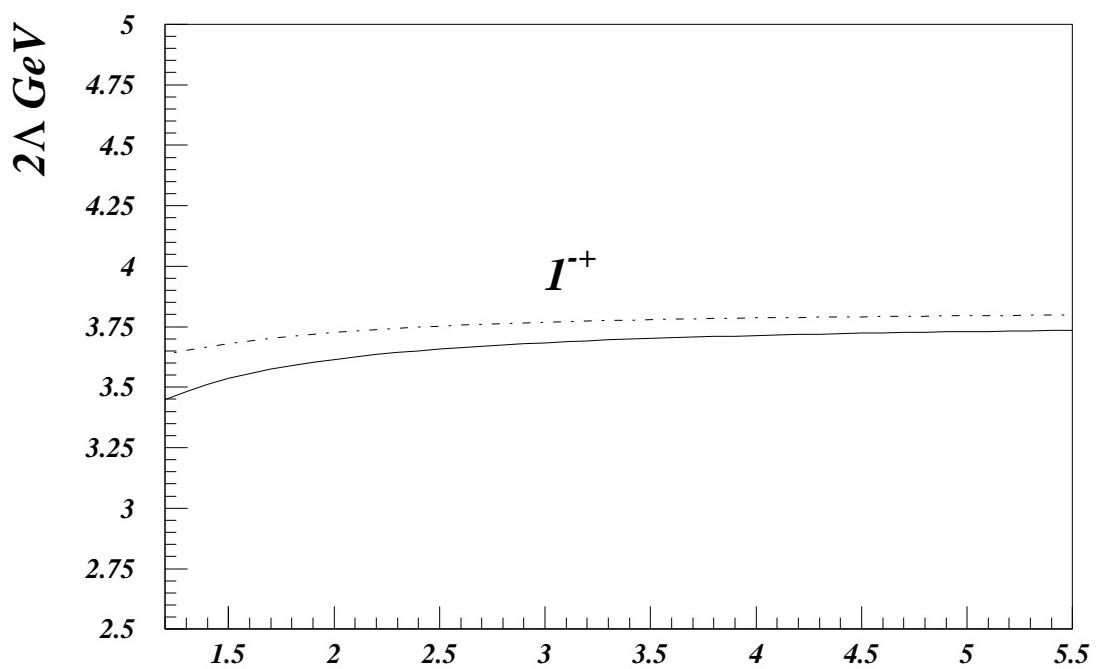
Fig. 4: Strong coupling of  $H'(0^{++})$  heavy-light hybrids versus Borel variable  $\tau$ .

Fig. 5: Strong coupling of  $H(1^{-+})$  heavy-light hybrids versus Borel variable  $\tau$ .

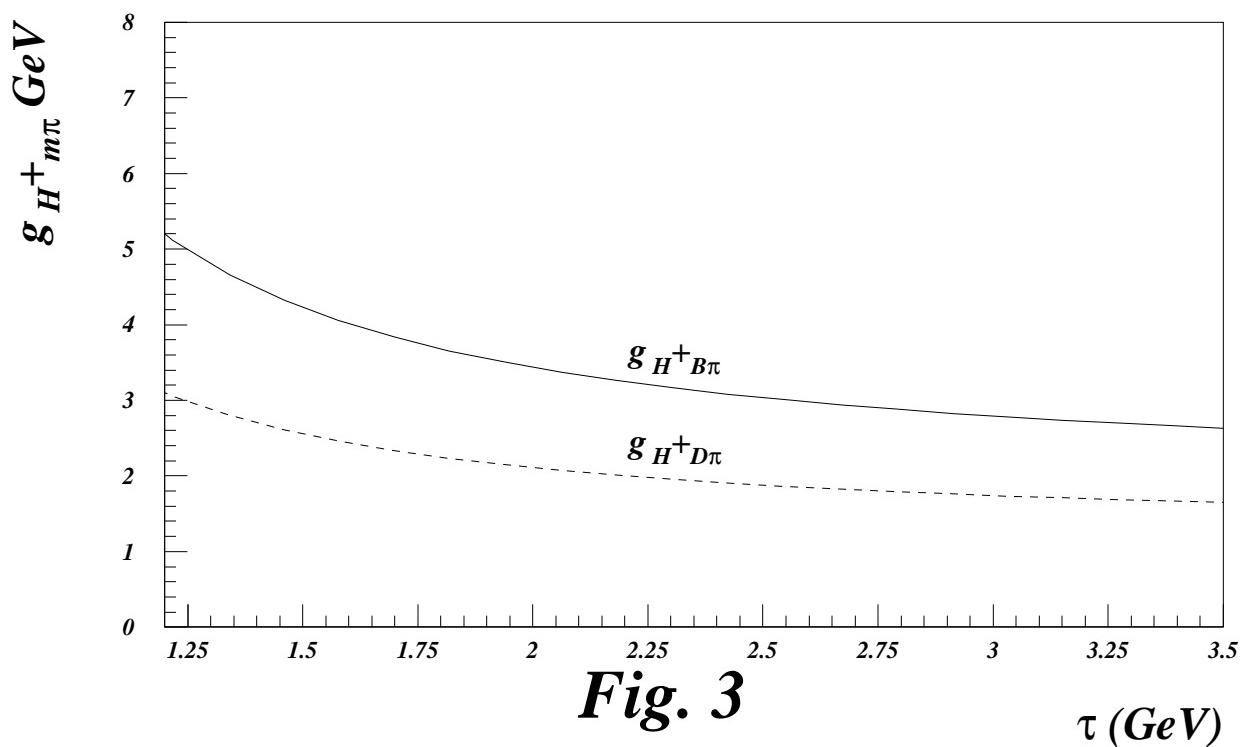


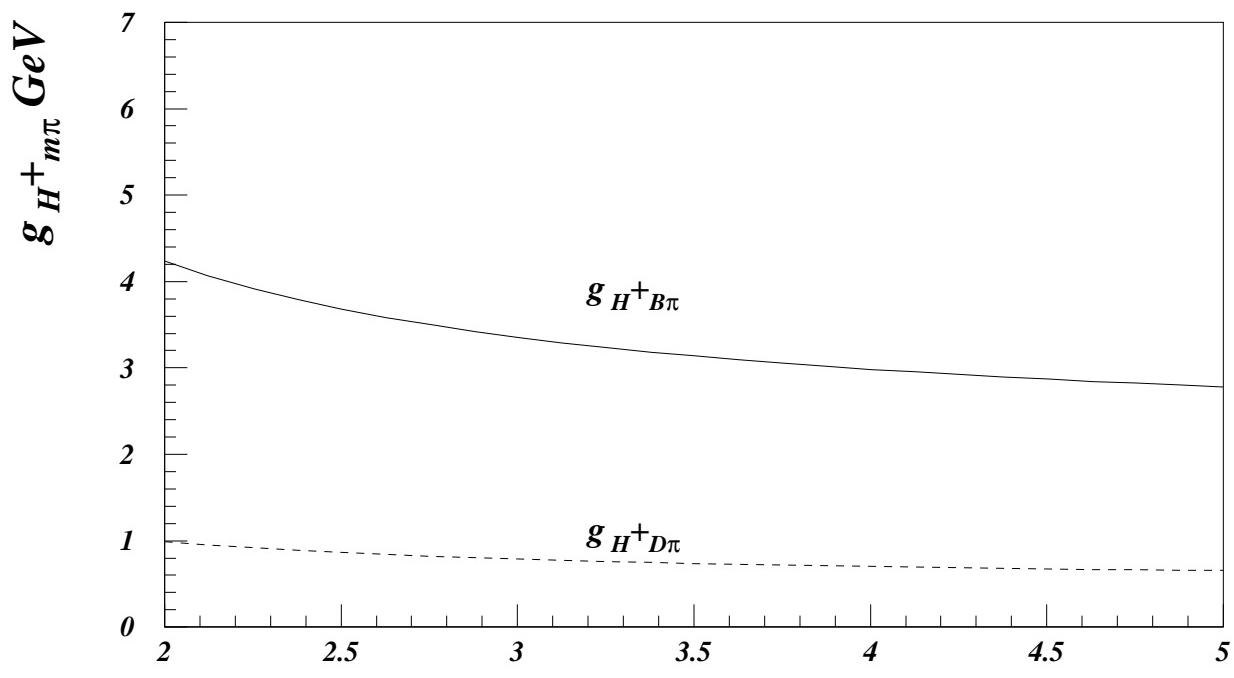
(e)

*Fig. 1*



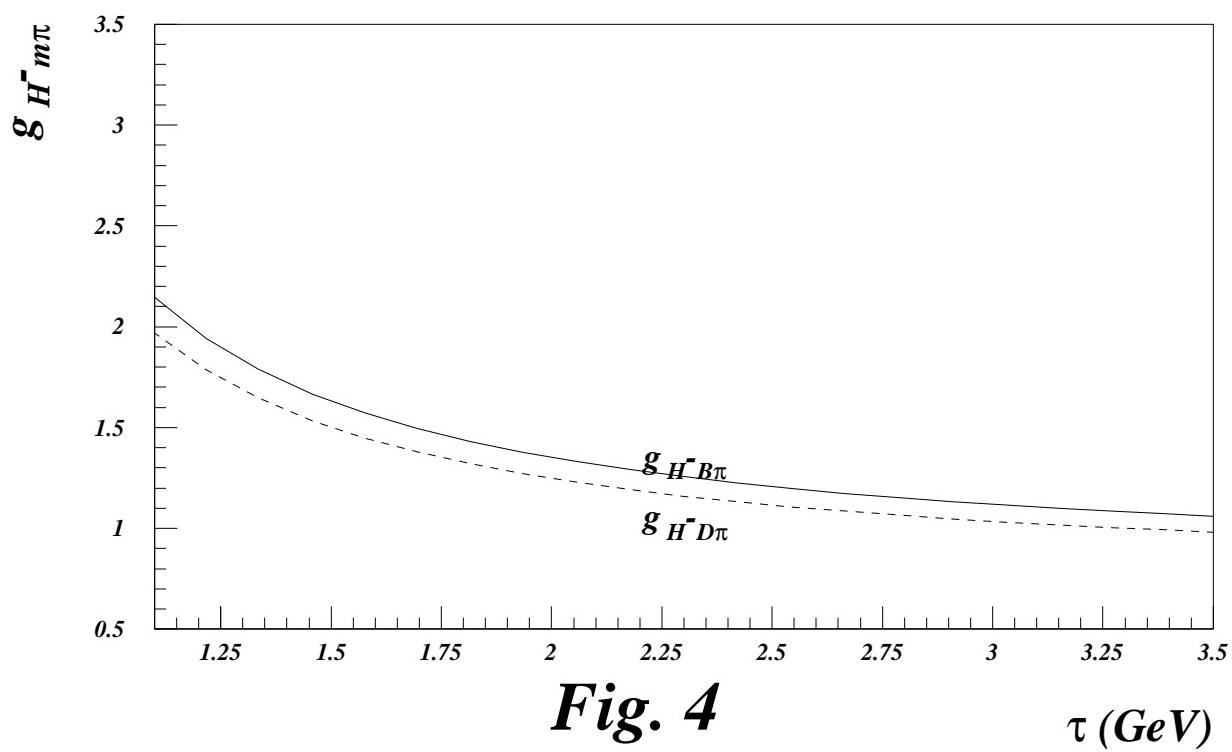
*Fig. 2*  $T$   $\text{GeV}$





**Fig. 4**

$\tau \text{GeV}$



**Fig. 4**